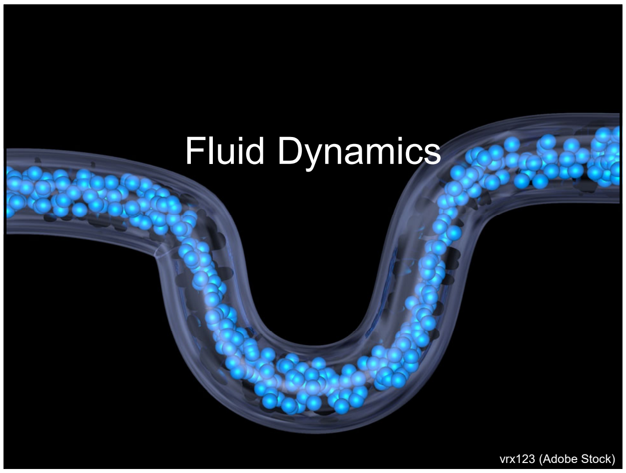


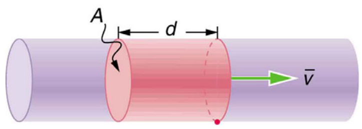
Flow Rate

- Flow rate Q, is defined to be the volume of fluid passing by some location through an area during a period of time.

$$Q = \frac{V}{t}$$

- Flow rate and velocity are related, but quite different, physical quantities.
  - If two rivers have the same width and depth, the slower moving one will have a smaller flow rate.
  - A wide and deep river will have a large flow rate even at slower speeds.
- We can derive an expression for the exact relationship between flow rate and velocity.

- Consider water flowing through a pipe of cross-sectional area, A.

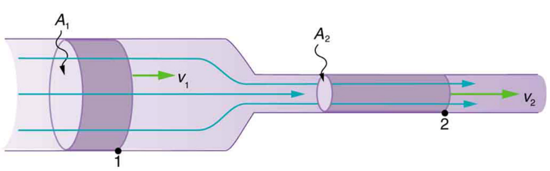


$$Q = \frac{V}{t} \qquad V = Ad$$

$$Q = \frac{Ad}{t} \qquad \bar{v} = \frac{d}{t}$$

$$Q = A\bar{v}$$

- Consider an incompressible fluid flowing along a pipe of decreasing radius.



- Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow.

- In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase.
  - This must be true for any points 1 and 2 along the tube.

$$Q_1 = Q_2$$

$$\boxed{A_1 v_1 = A_2 v_2}$$

- This is called the equation of continuity and is valid for any incompressible fluid.

Example 1

- A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water as it leaves the nozzle.

$$Q = A_{nozzle} v_{nozzle}$$

$$v_{nozzle} = \frac{Q}{A_{nozzle}}$$

$$v_{nozzle} = \frac{(0.5 \text{ L/s})(10^{-3} \text{ m}^3/\text{L})}{\pi(0.25 \times 10^{-2})^2}$$

$$v_{nozzle} = 25.5 \text{ m/s}$$

Example 2

In humans, blood flows from the heart into the aorta, then arteries, and eventually into a myriad of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm and the blood passes through it at a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4} cm and blood flows through with a speed of about 5×10^{-4} m/s. Estimate the number of capillaries in the human body.

$$A_1 v_1 = A_2 v_2$$

$$\pi r_{aorta}^2 v_1 = N \pi r_{capillaries}^2 v_2$$

$$N = \frac{r_{aorta}^2 v_1}{r_{capillaries}^2 v_2}$$

$$N = \frac{(1.2 \times 10^{-2})^2 (0.4)}{(4 \times 10^{-6})^2 (5 \times 10^{-4})}$$

$$N = 7 \times 10^9$$

Bernoulli's Equation

- When a fluid flows into a narrower channel, its speed increases.
- That means its kinetic energy also increases.
- Where does that change in kinetic energy come from?
  - The increased kinetic energy comes from the net work done on the fluid to push it into the channel.

- There is a pressure difference when the channel narrows.
- This pressure difference results in a net force on the fluid.
- The net work done increases the fluid's kinetic energy.
- As a result, the **pressure will drop in a rapidly-moving fluid**, whether or not the fluid is confined to a tube.

- Daniel Bernoulli, Swiss (1700-1782) worked out a principle concerning fluids in motion and developed an equation that expresses this principle quantitatively.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

ρ – fluid density
v – speed of fluid
g – gravitational field strength
y – the height above a chosen level
P – the pressure at the height y

- Bernoulli's equation is an expression of conservation of energy.

Example

- Water circulates through a house in a hot water system. The water enters the house with a speed of 0.50 m/s through a 4.0 cm diameter pipe with a pressure of 3.0×10^5 Pa. Calculate the pressure in a 1.0 cm diameter pipe on the second floor 5.0 m above. Assume the pipes do not divide into branches.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

First, we need to calculate speed of water on the second floor.

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi(0.02)^2(0.5)}{\pi(0.005)^2} = 8\ \mathrm{m/s}$$

Now we can calculate the pressure.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

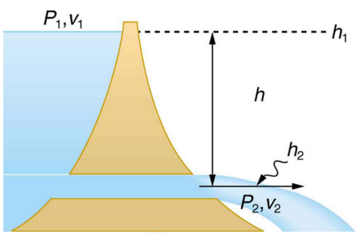
$$P_2 = P_1 + (\rho g y_1 - \rho g y_2) + \left(\frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2\right)$$

$$P_2 = (3 \times 10^5) + 1000(9.8)(0-5) + \frac{1}{2}(1000)\left((0.5)^2 - (8)^2\right)$$

$$P_2 = 2.2 \times 10^5\ \mathrm{Pa}$$

Example 2

Water rushes out of a channel through a dam. Derive an equation for the velocity of the water as it exits the channel.



$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Both P_1 and P_2 are open to the atmosphere and therefore are atmospheric pressure.

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 = \rho g y_2 + \frac{1}{2}\rho v_2^2$$

The fluid is incompressible, so the densities are all the same.

$$g y_1 + \frac{1}{2} v_1^2 = g y_2 + \frac{1}{2} v_2^2$$

Solve for v_2^2 and substitute in for the heights.

$$v_2^2 = v_1^2 + 2g(h_1 - h_2)$$

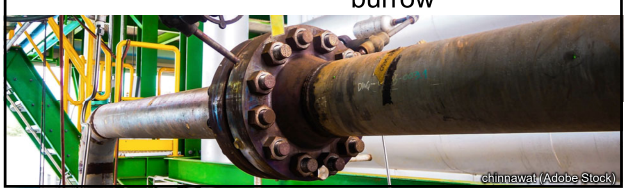
Replace $h_1 - h_2$ with h to represent the height the water drops.

$$v_2^2 = v_1^2 + 2gh$$

This is simply a kinematic equation for any object falling a distance h with negligible resistance. In fluids, this equation is called **Torricelli's theorem**.

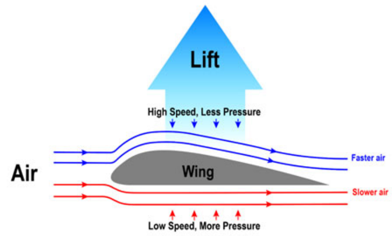
Other Applications of Bernoulli's Principle

- Airplane wing
- Venturi tube
- Pitot tube
- Baseball
- Transient Ischemic Attack (TIA)
- Entrainment
- Underground burrow

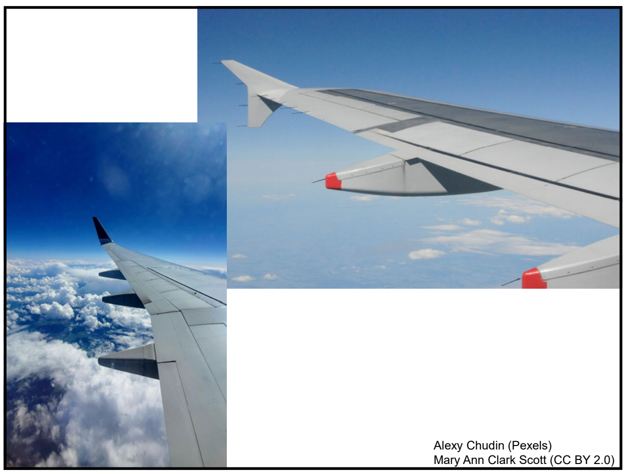

chinnawat (Adobe Stock)

Airplane Wing

- The velocity of air over the top of the wing is greater than under the wing causing a pressure difference.



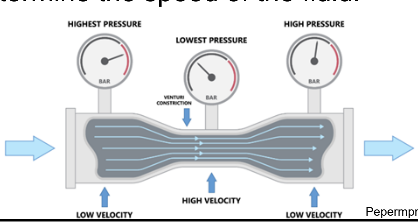
Saint Images (Adobe Stock)



Alexy Chudin (Pexels)
Mary Ann Clark Scott (CC BY 2.0)

Venturi Tube (Meter)

- The fluid speeds up in the narrow part causing a pressure change.
- The pressure differences are used to determine the speed of the fluid.

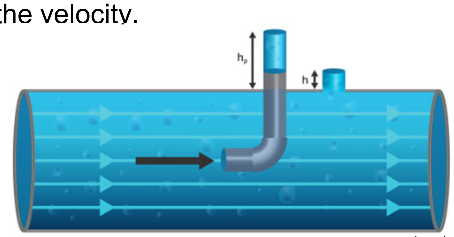


Pepermpron (Adobe Stock)

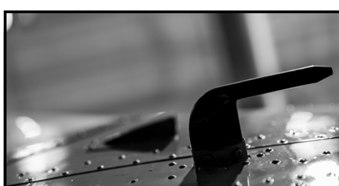

Glenelg Sewage Treatment Works (1930) – SA Water (CC BY-NC-ND 2.0)
Bristol Bolingbroke Mk. IV (Blenheim) (1942) – Kelly Michals (CC BY-NC 2.0)

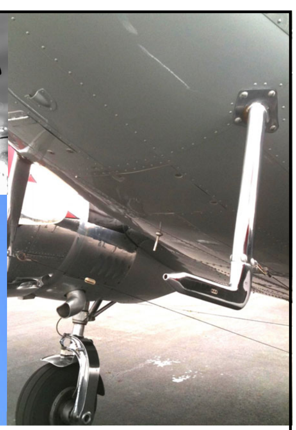
Pitot Tube

- The pressure difference between static pressure and total pressure (due to the movement of the fluid) is used to calculate the velocity.



petrroudny (Adobe Stock)

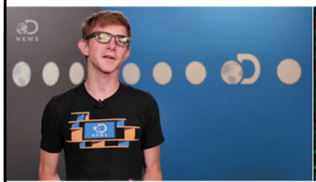


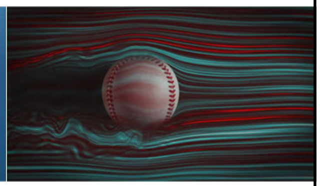


Pitot tube – Lost_in_the_Midwest (Adobe Stock)
Pitot tubes – Brave Heart (CC BY-NC-ND 2.0)
Pitot tube (Lockheed 12A Electra, 1939) – Brave Heart (CC BY-NC-ND 2.0)

Baseball

- The rotation of the baseball causes the air to move faster on one side, resulting in a change in pressure.

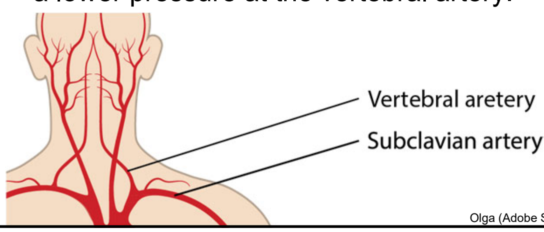




https://youtu.be/H09TIJ4a4fg?si=6e3xCqmeG7ZutXzL
https://youtu.be/hohkfrC-6nk?si=LFRpDQkGdbU5CHcu

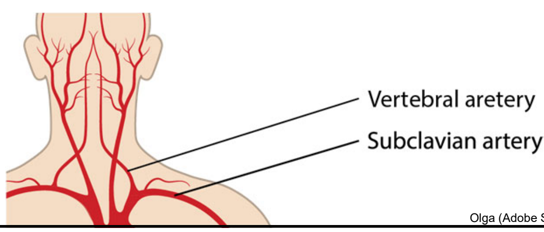
Transient Ischemic Attack (TIA)

- A blockage in the subclavian artery on one side will cause the velocity of the blood on that side to increase, resulting in a lower pressure at the vertebral artery.



Olga (Adobe Stock)

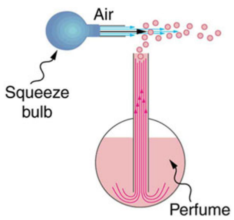
- This may cause the blood flowing up the unblocked side to be diverted into the vertebral artery instead of traveling to the brain.



Olga (Adobe Stock)

Entrainment

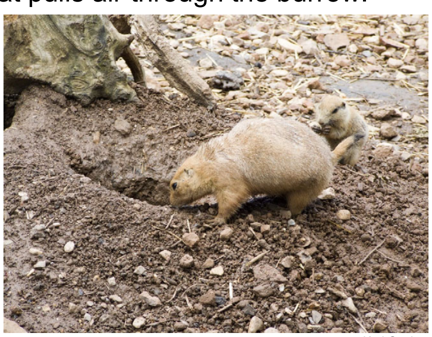
- A fast-moving fluid creates an area of high pressure that forces other fluids into the stream.



Underground Burrows

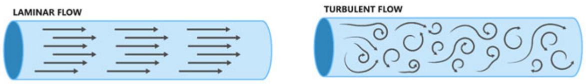
- Animals that live underground build burrows with at least two different entrances at different heights.

- Air that passes over the higher one is faster, creating an area of low pressure that pulls air through the burrow.



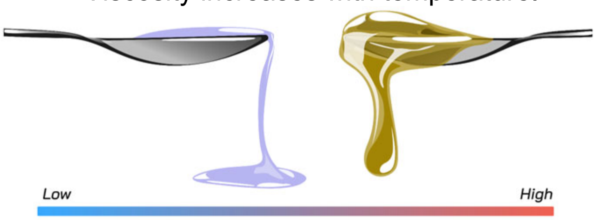
Laminar Flow and Viscosity

- Laminar flow is characterized by the smooth flow of the fluid in layers that do not mix.
- Turbulent flow, or turbulence, is characterized by eddies and swirls that mix layers of fluid together.



Pepermpron (Adobe Stock)

- Viscosity is fluid friction, both within the fluid itself and between the fluid and its surroundings.
  - Water has a low viscosity and syrup has a high viscosity.
  - Viscosity increases with temperature.



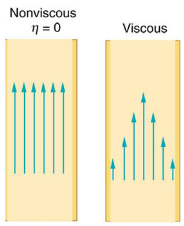
Anshuman Rath (Adobe Stock)

- Flow rate Q is in the direction from high to low pressure.
  - The greater the pressure differential between two points, the greater the flow rate.

$$Q = \frac{\Delta P}{R}$$

- The resistance R includes everything, except pressure, that affects flow rate.
  - For example, length of the tube, viscosity, turbulence, and diameter of the tube.

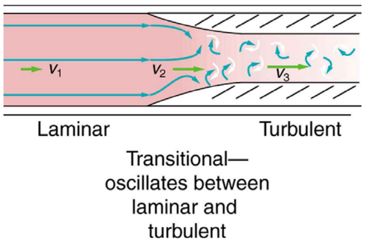
- If viscosity is zero, the fluid is frictionless and the resistance to flow is also zero.
- Comparing frictionless flow in a tube to viscous flow, for a viscous fluid, speed is greatest at midstream because of drag at the boundaries.



- The shape of the flame of a blow torch or a lighter is due to the viscosity of the gas.

Turbulence

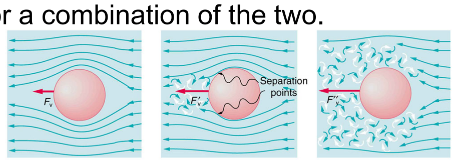
- Flow in a very smooth tube or around a smooth, streamlined object will be laminar at low velocity.
- At high velocity, even flow in a smooth tube or around a smooth object will experience turbulence.
- At intermediate velocities, flow may oscillate back and forth indefinitely between laminar and turbulent.



- An indicator called the Reynolds number can reveal whether flow is laminar or turbulent.

Motion of an Object in a Viscous Fluid

- A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream.
- Flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two.



- One of the consequences of viscosity is a resistance force called viscous drag.
- This force typically depends on the object's speed.